Planar quark scattering at strong coupling and universality

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ABSTRACT: We discuss scattering of fundamental matter in the planar and strong coupling limit via the AdS/CFT correspondence, generalizing the recently proposed calculation for adjoint matter due to Alday and Maldacena [1]. Color decomposition of quark amplitudes is a key property allowing to repeat the procedure in the case of fundamental matter and to derive the relation of these strong coupling amplitudes to minimal area problems. We present the results for two different $D3 - D7$ systems, one is only conformal in the planar limit and the other is exactly conformal. Our results suggest a universal behavior of scattering amplitudes at strong coupling and planar limit (both for gluons and quarks).

Contents

1. Introduction and summary

In the last decade great amount of research was devoted to the study of the AdS/CFT correspondence [2]. This correspondence provides us, among other things, with a technique to calculate correlators of composite gauge invariant operators in strongly coupled quantum field theories. On one side of the correspondence we have a gauge theory and on the other side a closed string theory, which is usually considered in the supergravity approximation. A priori any knowledge of correlators of the basic gauge fields is lost on the string theory side of the correspondence, as these are associated with open string degrees of freedom absent in a closed string theory.

Recently, a very interesting proposal has been put forward by Alday and Maldacena [1] to recover the information about correlators of the elementary gauge fields from the gravity dual.[1] The main input into the success of this program is that these correlators of gluons can be decomposed as products of tensors in the color space, and gauge invariant parts (in the sense that null states decouple) referred to as the "reduced amplitudes". This fact goes under the name of color decomposition.

Based on this, Alday and Maldacena have suggested a concrete way to compute gluon scattering amplitudes of strongly coupled planar $\mathcal{N} = 4$ SYM theory. In this limit, the scattering amplitudes are shown to be equivalent to computations of areas of surfaces whose boundary consists of light-like segments. The striking success of this prescription is the agreement of the result[2] with a conjectured form of these amplitudes due to Bern, Dixon and Smirnov [5] (see also [6]). Thus, it is a strong evidence in favor of this conjecture holding for all values of the coupling.

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[1] See [3] for more developments.

[2] In the case of 4-pt scattering, [1] have used a conformal transformation of a previously known solution due to [4] in order to obtain an explicit minimal surface.

In this note we discuss the planar contribution to the strong coupling quark scattering amplitudes in two different $\mathcal{N} = 2$ supersymmetric gauge field theories. The first is $\mathcal{N} = 4$ U(N) gauge theory deformed by adding an $\mathcal{N} = 2$ hypermultiplet in the fundamental representation [7]. The second is a conformal $\mathcal{N} = 2$ theory with a symplectic gauge group [8].

The results of [1] suggest universality of gluon scattering in a certain class of strongly coupled large N conformal field theories. This class consists of conformal field theories dual to a semiclassical string theory background of the form $AdS_5 \times W$. In this case, it is quite straightforward to repeat the procedure of [1], obtaining the same result.[3] It is unlikely that all these conformal theories have the same planar scattering amplitudes perturbatively in the 't-Hooft coupling (if this limit exists);[4] the universality occurs in strong coupling.

It is then natural to inquire whether similar universality holds for other matter fields which may appear in such theories (either exactly conformal theories or conformal only for large N). We analyze the case of fundamental representations in this note, and again find the same results for a-priori different strongly coupled planar theories. It is intriguing to understand from the field theory point of view why this universality occurs.

We wish to emphasize that neither gluon nor quark scattering is a well defined observable due to the IR divergences. However, if one takes into account gluon emission from the external lines then both become IR safe. It is, nevertheless, interesting to compute the strict n point function as it is a building block in many IR safe computations.

This note is organized as follows. In section 2 we describe the way color decomposition works in the case of fundamental matter. In section 3 we review how one can add quarks to $\mathcal{N} = 4$ theory. In section 4 we repeat the necessary steps for calculating the reduced amplitude of quarks scattering in the model of section 3. In section 5 we discuss another model with "quark" fields and repeat the argument once again.

2. Color decomposition of quark scattering

Let us briefly review color decomposition of gluon amplitudes [10, 11] (see [12] for a review). The general amplitude depends on the color indices, momenta and helicities of each in-going gluon (we assume all of them are in-going for simplicity). These quantum numbers are denoted by a_i, k_i, h_i respectively. It can be shown that planar amplitudes factorize as

$$\mathcal{M}_n^{\text{gluons}}(k_1, h_1, a_1; k_2, h_2, a_2; \ldots; k_n, h_n, a_n) = \tag{2.1}$$
$$= \sum_{\sigma \in S_n/\mathbb{Z}_n} Tr(T^{a_{\sigma(1)}} T^{a_{\sigma(2)}} \ldots T^{a_{\sigma(n)}}) A_n^{\text{gluons}}(k_1, h_1; \ldots; k_n, h_n),$$

where the sum is over permutations which do not differ by cyclic permutations. In this paper, we consider U(N_c) gauge theory with minimally coupled quarks in the fundamental

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[3]In [1], the classical solution does not involve the S^5 as it can only increase the action. This is why one expects the result to be independent of the transverse space.

[4]One has to keep in mind that there are possibly many planar equivalences (for a review see [9]).
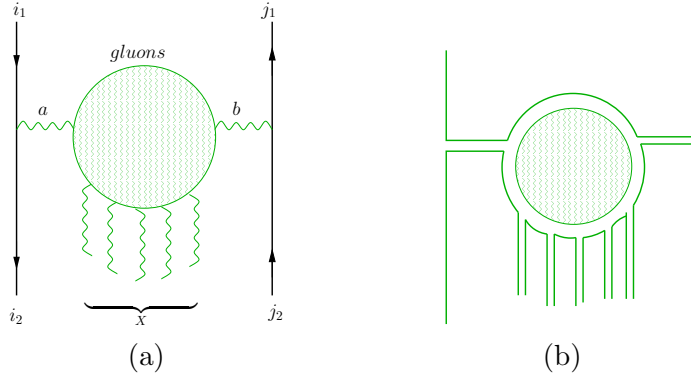
Figure 1: Example of quark scattering diagram, (a) the Feynman diagram (b) its double line notation.

representation and anti-quarks in the anti-fundamental representation (there may also be couplings to adjoint matter). Hence, one should first generalize (2.1) to this case [13, 14].[5]

For simplicity, we assume that there is only one flavor (this assumption can be easily generalized). For the combinatorial analysis, we also assume that all the momenta are ingoing. Thus, we actually describe scattering of n quarks and n anti-quarks. One would like to calculate the connected amplitude for such a scattering process

$$\mathcal{M}_n^{\text{quarks}}(k_1, \epsilon_1, a_1; k_2, \epsilon_2, a_2; \ldots; k_n, \epsilon_n, a_n | r_1, \bar{\epsilon}_1, \bar{a}_1; r_2, \bar{\epsilon}_2, \bar{a}_2; \ldots; r_n, \bar{\epsilon}_n, \bar{a}_n), \qquad (2.2)$$

where k_i, r_i are the momenta of the quarks and the anti-quarks respectively, $\epsilon, \bar{\epsilon}$ are their respective polarizations and a, $\bar{a}$ are their respective color indices. This is, of course, a formidably hard problem. We would like to show that there is a very convenient way of taking into account the color dependence of planar diagrams, analogous to (2.1). The first simplification due to the planar limit is that quark loops are sub-leading in this approximation. Hence, if a propagator in a Feynman diagram is not a "continuation" of an external quark propagator it must be an adjoint field. Let us abbreviate (2.2) as

$$\mathcal{M}_n^{\text{quarks}}(1; 2; \ldots; n | \bar{1}; \bar{2}; \ldots; \bar{n}). \qquad (2.3)$$

To understand the color flow it is best to think in 't Hooft's double line notation, but let us do an example explicitly. Consider the diagram depicted in figure 1(a). The indices are color indices, and the arrows are the color flow direction consistent with planarity.

The color structure of this amplitude is given by

$$\sum_{a,b} T_{i_1 i_2}^a \mathcal{M}_n^{\text{adjoint}}(k_1, h_1, a; k_2, h_2, b; \ldots) T_{j_2 j_1}^b. \qquad (2.4)$$

Adjoint amplitudes color decomposition simplifies the color dependence of the expression above to

$$\sum_{a,b,X} T_{i_1 i_2}^a Tr(T^a T^b X) T_{j_2 j_1}^b, \qquad (2.5)$$

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[5]A more general problem includes quarks, anti-quarks and gluons as external states. The color structure in this case can be successfully analyzed as well - for a review see [12] and references therein.

where X encompasses all the color information associated to the emitted gluons other than a and b. Finally, we can use some simple U(N_c) identities in a convenient normalization

$$Tr(T^a T^b) = \delta^{ab},$$

$$\sum_{a=1}^{N_c^2} T_{ij}^a T_{kl}^a = \delta_{il}\delta_{jk}, \tag{2.6}$$

where T^a's are $n \times n$ hermitian matrices to get

$$\sum_{a,b,X} T_{i_1 i_2}^a Tr(T^a T^b X) T_{j_2 j_1}^b = \delta_{i_1 j_1} \sum_X (X)_{i_2 j_2}. \tag{2.7}$$

The important term is $\delta_{i_1 j_1}$. Drawing the diagram in double line notation, as in figure 1(b), this delta function is obvious since there is a line along which the color i_1 flows, and eventually connects to j_1. Similarly, for each Feynman diagram one should follow the flow of the external color. This pairs the quarks and the anti-quarks in a unique way and gives a delta function for each such pair. Consequently, the natural color decomposition of (2.3) is

$$\mathcal{M}_n^{\text{quarks}}(1;2;\ldots;n|\bar{1};\bar{2};\ldots;\bar{n}) = \sum_{\sigma \in S_n} \delta_{a_1 \bar{a}_{\sigma(1)}} \cdots \delta_{a_n \bar{a}_{\sigma(n)}} A_n^{\text{quarks}}(1;\ldots;n|\overline{\sigma(1)};\ldots;\overline{\sigma(n)}). \tag{2.8}$$

The indices of the function A_n^{quarks} stand for momenta and polarization quantum numbers only (it is color independent by construction). Two remarks are in order. First, the result (2.8) is a very general one. A slight generalization of the construction above implies that the set of tensors spanned by

$$\delta_{a_1 \bar{a}_{\sigma(1)}} \cdots \delta_{a_n \bar{a}_{\sigma(n)}} \tag{2.9}$$

is a suitable set of tensors to expand any amplitude of quark scattering, even beyond the planar limit.[6] Intuitively, the reason is that following external fundamental color line in the double line notation, even for a diagram of some non zero genus, eventually leads one to an external anti-quark producing the required delta function. However, this does not mean that the expansion (2.8) is *useful* beyond the planar limit. The point is that the reduced amplitude, A_n, is not guaranteed to be gauge invariant in general (in the sense of null states decoupling). In the planar limit, one can prove that reduced amplitude are gauge invariant by utilizing the approximate orthogonality of the tensors (2.9) (for details see [12]). This guarantees their linear independence in leading order.

3. Adding fundamental matter to $\mathcal{N} = 4$

We wish to add fundamental matter to $\mathcal{N} = 4$ so that we can study its scattering amplitudes. We briefly review here two ways in which quarks can be added to $\mathcal{N} = 4$ in the framework of AdS/CFT. Another way to do it is explained in section 5.

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[6]In the case of gluon scattering, the set of tensors spanned by the single traces is not sufficient beyond the planar limit. One needs to include multi-traces as well.
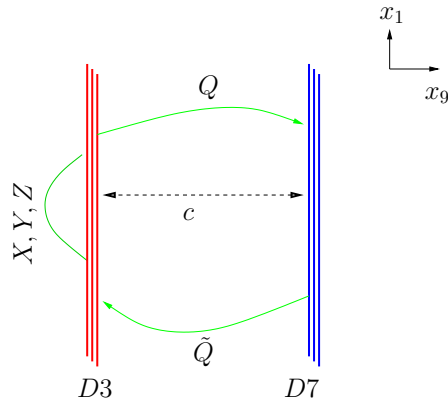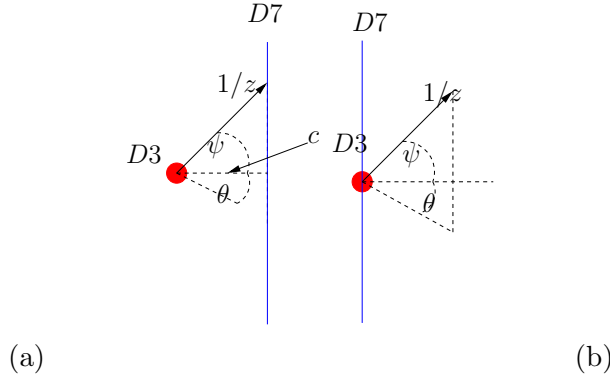
Figure 2: The $D3 - D7$ system.

The simplest way to add quarks is by going to the Higgs branch and breaking $SU(N + M)$ to $SU(N) \times SU(M)$. We take N large but M finite, and take the near horizon limit of the N branes considering the remaining M branes in the probe approximation. No SUSY is broken and the fundamental matter comes from strings stretching between the probe D-branes to the IR region in the near-horizon geometry. The fundamental matter resides in the vector multiplet of $\mathcal{N} = 4$, it is in the fundamental/antifundamental representation of the $SU(N)$ group and in the anti-fundamental/fundamental representation of the $SU(M)$ flavor group. The mass of the quarks is proportional to the separation between the two stacks of D-branes. Note that when we take the separation to zero the gauge group becomes $SU(N + M)$, and the fundamental matter of $SU(N)$ becomes part of the adjoint of the $SU(N + M)$ group. Thus, in this limit massless quarks have by construction essentially the same properties as gluons.

The second and more interesting way to add fundamental matter is by adding M $D7$-branes to N $D3$-branes [7].[7] Again, we take N large but M finite. SUSY is broken to $\mathcal{N} = 2$. The fundamental matter comes from strings stretching between the $D7$-branes and the IR region of the near-horizon geometry. These fields sit in the hypermultiplet of $\mathcal{N} = 2$ and transform in the fundamental/anti-fundamental representation of the $SU(N)$ group and in the anti-fundamental/fundamental representation of the $SU(M)$ flavor group. The mass of the quarks is given by the separation of the $D7$-branes from the $D3$-branes (see figure 2). In the near horizon limit the $D7$ branes fill the AdS_5 part of the space up to a specific IR cutoff given by the mass of the quarks. We will be interested in massless quarks in what follows.

Let us specify some details of this theory. We take the stack of M $D3$-branes to span the directions $x_{0,...,3}$. The stack of M $D7$-branes is sitting at $x_8 = 0$ and $x_9 = c$ and spans the remaining space-time directions. Note that the mass of the quarks is proportional to

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[7]See also [15] for more details.

Figure 3: The $D3 - D7$ system before backreaction considered. (a) The massive case, here the $D7$ wraps the sphere in a non-trivial way, $\cos(\psi) = c \cdot z$. (b) The massless case, the $D7$ brane lies along the $\psi = \frac{1}{2}\pi$ direction, and spans all values of z.

the parameter $\frac{c}{l_s^2}$. Further, we define

$$\frac{1}{z^2} \equiv \sum_{i=4}^{9} x_i^2, \tag{3.1}$$

which becomes the radial direction of the AdS_5 factor and the sub-manifolds of constant z span the S^5 part of the the near horizon geometry. The metrics on the AdS_5 and the S^5 factors are given by,

$$ds_{\mathrm{AdS}}^2 = \frac{R^2}{z^2}\left(dz^2 + \sum_{i,j=0}^{3} \eta^{ij} dx_i dx_j\right), \quad ds_{S^5}^2 = R^2\left(d\psi^2 + \cos^2(\psi)d\theta^2 + \sin^2(\psi)d\Omega_3^2\right), \tag{3.2}$$

where the $D7$-branes wrap the S^3 factor as

$$\cos(\psi) = c \cdot z. \tag{3.3}$$

The geometrical setup giving rise to (3.3) is depicted in figure 3(a)

Note that for non zero c the $D7$-branes fill the AdS_5 space for $z \leq 1/c$, and only for $c = 0$ the whole AdS space is covered (this special case is depicted in figure 3(b)). We will specify now to the massless case by taking $c = 0$. The superpotential in this case is given by

$$W = X\left[Y, Z\right] + YQ\tilde{Q}, \tag{3.4}$$

where Q and $\tilde{Q}$ form together a hypermultiplets of $\mathcal{N} = 2$ coming from strings stretching between the $D7$ and the $D3$ branes. Note that the fundamental matter couples to a single $\mathcal{N} = 1$ adjoint scalar Y. The choice of this scalar is dictated by the choice of the S^3 inside the S^5 which the $D7$-branes wrap, given in our case by (3.3).

4. n-point quark scattering

In this section we explain the minimal area problem which is relevant for computing quark amplitudes at strong coupling. We introduce quarks via the $D3 - D7$ system discussed above. The massless quarks come from strings connecting a probe $D3$ brane located at $z = \infty$ and the probe flavor $D7$ branes. The $D7$ branes span the whole AdS factor and are localized at $\psi = \frac{1}{2}\pi$ on the sphere. We regulate the possible IR divergences with dimensional regularization. The dimensionally continued metric of $p = 3 - 2\epsilon$ branes is given by (the r coordinate is given by $r = \frac{R^2}{z}$) [1]

$$
ds^2 = \frac{r^2 dx_D^2}{\sqrt{\lambda} \left[2\pi \frac{\mu}{r} e^{\tilde{\gamma}}\right]^{\epsilon} \sqrt{\Gamma(2+\epsilon)}}
$$
$$
+ \sqrt{\lambda} \left[2\pi \frac{\mu}{r} e^{\tilde{\gamma}}\right]^{\epsilon} \sqrt{\Gamma(2+\epsilon)} \frac{dr^2}{r^2} + \sqrt{\lambda} \left[2\pi \frac{\mu}{r} e^{\tilde{\gamma}}\right]^{\epsilon} \sqrt{\Gamma(2+\epsilon)} d\Omega_{9-D}^2,
$$
$$
\tilde{\gamma} = -\frac{1}{2}\Gamma'(1), \qquad \lambda = g^2 N, \qquad D = 4 - 2\epsilon. \tag{4.1}
$$

The relevant sign of the regulator is $\epsilon < 0$ (the reason is that IR divergences are regulated by increasing the dimension of space time), λ is the dimensionless $4d$ 't Hooft coupling, and μ is the IR scale of dimensional regularization.

In principal, one has to specify the position of the $D3$ brane on the S^5 since there is an invariant angle between the $D3$ and the $D7$. Irrespectively of this angle, because of the infinite rescaling of the energy

$$
E_{\mathcal{N}=4} = \frac{R}{z} E_{10D}, \tag{4.2}
$$

the quarks in the gauge theory are massless as long as the $D3$ sits at $z = \infty$. Thus, the (dimensionally regularized) scattering process we perform in field theory is independent of this angle, and so should be the final result we get from string theory. For simplicity we may put our $D3$ brane at $\psi = \frac{1}{2}\pi$, on top of the $D7$ branes.

We wish to emphasize that this angle independence is necessary for the consistency of the prescription[8] not only in the $D3 - D7$ system but also in the case of gluon scattering. This issues was not relevant in [1], but it certainly becomes relevant if one takes out more than a single $D3$ out of the stack. One may decide to scatter massless gluons connecting the two $D3$ branes which were removed from the stack. There must not be any angle dependence due to color decomposition in the field theory process.[9]

The main a priori difference between scattering of gluons and scattering of quarks is that the $D7$ branes are not localized along the z direction. This implies that the boundary of the disc on which the string vertex operators are inserted may not be restricted to $z = \infty$; it is a Neumann direction.

The power and the simplicity of the Alday-Maldacena mechanism is the translation of the scattering problem into a minimal area problem with a prescribed boundary. The

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[8] We are grateful to O. Aharony for invaluable discussions on this.

[9] It will be interesting to understand better how this comes about.

translation between these problems goes through a formal T-duality procedure of some of the AdS_5 coordinates (all except the radial one). The T-dual coordinates are defined to be

$$\partial_\alpha y^\mu = i\frac{r^2}{R^2}\epsilon_{\alpha\beta}\partial^\beta x^\mu. \tag{4.3}$$

The T-dual space has still an AdS_5 metric which has the following dimensionally regularized form

$$ds^2 = \sqrt{\lambda}\left[2\pi\mu e^{\tilde\gamma}\right]^\epsilon\sqrt{\Gamma(2+\epsilon)}\frac{dr^2+dy_D^2}{r^{2+\epsilon}} + \sqrt{\lambda}\left[2\pi\mu e^{\tilde\gamma}\right]^\epsilon\sqrt{\Gamma(2+\epsilon)}\frac{d\Omega_{9-D}^2}{r^\epsilon}. \tag{4.4}$$

After this T-duality the scattering problem translates to a minimal area problem with the boundary of the surface fixed by straight light-like segments.

We are looking for a classical solution of the world-sheet embedding into AdS space around an insertion of a quark vertex operator. Consider the following vertex operator

$$V \sim e^{ik^\mu x_\mu}, \qquad k^2 = 0, \tag{4.5}$$

where we have truncated to the lowest lying excitation in the x fields (in other words we have not included any function of the derivatives of x). The exponent is determined by shift symmetries in x and the AdS metric in the Poincare patch is given by (3.2). Thus, the classical action of the string with such a vertex operator is given by

$$S = \int d^2w \left[\frac{R^2}{2\pi\alpha'z^2}\left(\partial z\bar\partial z + \partial x^\mu\bar\partial x_\mu\right) + ik^\mu x_\mu\delta^{(2)}(w,\bar w)\right], \tag{4.6}$$

and the equations of motion are given by

$$\bar\partial\partial x^\mu - \frac{1}{z}(\partial z\bar\partial x^\mu + \bar\partial z\partial x^\mu) = i\frac{\pi\alpha'z^2}{R^2}k_\mu\delta^{(2)}(w,\bar w), \qquad \bar\partial\partial z + \frac{1}{z}\left(\partial x_\mu\bar\partial x^\mu - \partial z\bar\partial z\right) = 0. \tag{4.7}$$

Indeed, a solution to this coupled system can be easily found in the massless case

$$z = z_0 \qquad x_\mu \sim i\frac{\alpha'z_0^2}{2R^2}k_\mu\ln|w|^2. \tag{4.8}$$

It can be easily seen to satisfy the classical constraint $L_0 = 0$. Of course, up to now there is absolutely no technical difference between the Dirichlet-Dirichlet case considered in [1] and the Dirichlet-Neumann case we consider here. The solution (4.8) satisfies both types of boundary conditions, and describes a massless string which does not stretch in the z direction at all.

A difference does arise if we recall that the full scattering problem has many vertex operators, and the embedding along the boundaries connecting different vertex operators can be non trivial. In the case of [1] this is excluded automatically by the Dirichlet-Dirichlet boundary conditions, but not in our case. Remembering that near each vertex operator the consistent solution has constant z, a variation of z between different vertex operators results in a singular surface describing the boundary condition in the T dual frame. This situation is depicted in figure 4. The reason for these spike singularities in the boundary
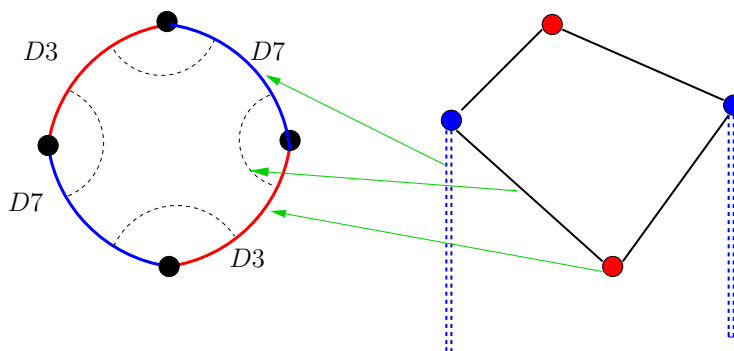
Figure 4: The disc four point amplitude before (left) and after (right) the action of the T-duality. The spike singularities which arise if z is allowed to vary along the blue lines on the left hand side figure, are depicted as blue dashed lines.

surface is that the T dual y's are constant along the boundary line connecting different vertex operators while the z's vary by assumption. The opposite situation occurs in the neighborhood of each vertex operator.

Thus, in the end, the calculation of the quark scattering at strong coupling necessarily amounts to a solution of a minimal area problem with fixed boundary formed by light-like segments, perhaps with a spike. After this paper has appeared several arguments were presented in [16] in favor of the solution with the spikes.[10]

5. $\mathcal{N} = 2$ conformal field theory

In this section we analyze a closely related case, the $\mathcal{N} = 2$ conformal field theory of [17, 8, 18].[11] It consists of $USp(2N)$ gauge theory with one hypermultiplet in the antisymmetric representation and four hypermultiplets in the fundamental representation. It can be realized by probing F theory [19] compactified on K3 at a special point in moduli space with $D3$ branes. At this special point, the K3 is actually the orbifold $\mathbb{T}^4/\mathbb{Z}_2$ [20]. In ten dimensions, in the vicinity of one of the four fixed points, this corresponds to a $\mathbb{Z}_2$ orientfold of IIB string theory in flat space which consists of an $O7^-$ plane and four $D7$ branes on top of it. This cancels the tadpole locally, allowing the string coupling to be arbitrary. $D3$ branes probing this configuration carry the $\mathcal{N} = 2$ theory described above where the fundamental hypermultiplets arise from D3-D7 strings. In the origin of the moduli space of D3 branes all the fields are massless and the theory is exactly conformal.

In the large N limit with large 't-Hooft coupling this is described by the dual IIB string theory background [21]

$$ds^2 = \frac{R^2}{z^2}\bigg(dz^2 + \sum_{i,j=0}^{3} \eta^{ij}dx_idx_j\bigg) + R^2\bigg(d\psi^2 + \cos^2(\psi)d\theta^2 + \sin^2(\psi)d\Omega_3^2\bigg), \qquad (5.1)$$

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[10]In a previous version of this paper we assumed that there should be no spikes. However, in light of the arguments presented in [16] the spikes should be taken into consideration.

[11]We are grateful to S.Yankielowicz who brought this model to our attention.

where the essential difference from the usual case is that θ has periodicity π instead of 2π and that the doublet of two forms $(B^{(2)}, C^{(2)})$ goes to $(-B^{(2)}, -C^{(2)})$ upon π rotation in θ. For this specific singularity, the axion-dilaton field doesn't have non trivial monodromies since the $SL(2, \mathbb{Z})$ element defining the fiber in F theory is actually an identity in $PSL(2, \mathbb{Z})$.

We would like to emphasize that the presence of dynamical quarks implies that the solution (5.1) actually contains in it $D7$ branes whose back-reaction was taken into account. In a sense, the background (5.1) should be thought of as already having $D7$ branes, and the angle θ should be thought of as the angular variable in the transverse space to the $D7$s which are localized in $\psi = \pi/2$, wrap the remaining S^3 and are extended in all the directions of AdS_5 (the situation is practically the same as in figure 3 only that one has to add an orientfold plane and the branes come in pairs).

For our purpose it remains to emphasize again that this theory contains dynamical fields in the fundamental representation, and that both what we have done above and the gluon scattering of [1] can be repeated *verbatim*. In particular, color decomposition holds in this case as well (with a small technical difference due to the fact that the fundamental representation is equivalent to its conjugate) and one can take a D3 (and its mirror which is reached after a rotation in θ) out of the stack, replacing the stack by the gravity background (5.1). Scattering of gauge bosons in this system corresponds to scattering in the $\mathrm{U}(1)$ of the coulomb branch of $USp(2N)$

$$USp(2N - 2) \times \mathrm{U}(1) \hookrightarrow USp(2N). \tag{5.2}$$

This branch may be reached by turning VEVs to the scalars in the vector multiplet or by turning VEVs of scalars in the antisymmetric hypermultiplet. The difference is that the latter moves the branes parallel to the $D7$ branes while the former moves them in transverse directions.[12] Since gluons in the $\mathrm{U}(1)$ are just Dirichlet-Dirichlet strings of the $D3$ brane, it does not matter where it sits on the orientfolded S^5 (as in the oriented case, there must be "angle independence" here as well) and the T duality of [1] is performed in exactly the same way, with the same result for the reduced gauge invariant amplitude of gluon scattering.

Scattering of fundamental hypermultiplets is very natural in this system, since no $D7$ branes have to be introduced by hand. On the coulomb branch (5.2) the hypermultiplets are massive and are described by Dirichlet-Neumann strings with an end on the $\mathrm{U}(1)$ $D3$ brane and the other end at any point in AdS.[13] To get the theory in the origin of the moduli space we bring the $D3$ brane back to the origin. It is clear that the minimal area problem we get here is the same as the one we got for the $\mathcal{N} = 2$ theory analyzed in the previous section, implying a universal result at strong coupling.

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[12]Note that if there is no movement at all in transverse directions to the $D7$, then other symmetry breaking patterns become possible, for instance $USp(2N - 2) \times USp(2)$. This subtlety will not change anything in our considerations.

[13]Of course, these strings must also stretch in the transverse space to AdS if we reached the coulomb branch with vector multiplet expectation values.

Acknowledgments

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